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GENDEX: NOA MODULE FOR CONSTRUCTING ORTHOGONAL AND NEAR-ORTHOGONAL ARRAYS AND MAIN EFFECT PLANS

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ABSTRACT

The NOA module of the Gendex toolkit may be used to obtain 2-level main effect plans for 2 to 128 factors, for constructing orthogonal or near-orthogonal arrays, and for constructing supersaturated fractional replicates of a complete factorial. In addition, a 2.level main effect plan may be augmented with single degree of freedom contrasts for a k-level factor with this module. An explanation of the steps to construct a plan is given. Several examples are presented to illustrate various features of the NOA module.

Keywords: Supersaturated designs, Helmert contrasts, Orthogonal polynomial contrasts, Single degree of freedom contrasts, Concurrences, Fractional replicate.

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INTRODUCTION

NOA is a program for constructing orthogonal and near-orthogonal arrays with mixed leveled factors. These plans can also be considered as 2-level main effect plans for fractional replicates of a complete factorial for $2 \leq m \leq 128$ factors in $2 \leq n \leq 128$

runs. This program is also useful for constructing supersaturated treatment designs (See Booth and Cox, 1962; Lin, 1993). A supersaturated treatment design is a near-orthogonal array with m 2-level factors in n runs $L'_n(2^m)$ with $m > n - 1$. Also, these plans may be augmented with single degree of freedom contrasts for k-level factors. The optimality criterion and the algorithm used in NOA are described in Nguyen (1996a, 1996b). Two criteria are options for constructing a design. The first option (Booth and Cox, 1962) minimizes the average value of the squared values of the concurrences in the information matrix. The second option (Nguyen, 1996a, 1996b) minimizes the maximum value of the frequency of the largest concurrence in the information matrix and is denoted as minimax.

STEPS TO OBTAIN AN ARRAY WITH NOA

We shall assume that the Gendex toolkit is placed in the C:\ drive and in the folder named Gendex as described by Federer *et al.* (2001). The following are the steps to obtain a desired array:

1. Click on START/PROGRAMS/MS_DOS PROMPT.
2. Change the directory with CD\Gendex command to be in the C:\Gendex> mode.
3. Type NOA (case is unimportant), return.
4. A screen appears with the statement "Choose the number of 2-level factors:". The number of factors m may be between 2 and 128. Select appropriate m and click OK.
5. A screen will appear with the statement "Choose the number of runs:". The number of runs n may be between 3 and 128. Select an appropriate n and click OK.
6. A screen will appear with the statement : "Choose a criterion:" The two choices are Booth-Cox E(s^2) and Minimax.
7. A screen will appear with the statement "Enter a random seed:". A number may be entered or the space may be left blank. Click on OK.
8. A screen will appear with the statement "Enter number of tries:". A number may be entered or the space may be left blank. Click on OK.
9. An output of the selected design and its properties appear on the screen. Click OK at the bottom of the screen to save as NOA.HTM in the C:\Gendex directory.
10. A screen will appear describing the NOA.HTM just saved. Also, the cursor is at the C:\Gendex> place on the screen and another design may now be obtained if desired.
11. A printed output of NOA.HTM may be obtained by going to C:\Gendex in WINDOWS EXPLORER. Highlight the file NOA.HTM. Under EDIT, click on SELECT ALL and then on PRINT under FILE.

PRINTED OUTPUT

The results of the NOA session will be reported on the computer screen and be saved in the NOA.HTM file. Information for the various tries includes:

1. There is a note to the effect that m = number selected and n = the number selected.

2. The random seed number selected by user or by the computer time clock.
3. The try number.
4. The number of iterations for each try.
5. For s_{ij} equal to the element (concurrence) in the ith row and jth column of the information matrix **X'X**, the average value of the s_{ij}^2 above the main right diagonal is computed as $2\sum_{i<j} s_{ij}^2 / m(m - 1)$ = ave. If an asterisk appears on this line, it means this quantity has reached its lower bound.
6. The maximum value of the absolute value of any s_{ij} above the main right diagonal is recorded as max of s(ij).
7. The frequency of the maximum value is recorded as freq of max.
8. The D-optimal efficiency is given as D-eff and is computed as the determinant of the m^{th} root of the correlation matrix.
9. The A-efficiency is given as A-eff. This value is computed as the determinant of the correlation matrix or as the determinant of the **X'X** matrix divided by the product of the diagonal elements of **X'X**.
10. Under Factor levels, the best treatment design found for all the tries used is given. This is the **X** matrix.
11. The elements in the main right diagonal and those above of the **X'X** matrix are given.
12. The correlation matrix, **C**, is given next. This describes the correlation between the columns of **X**.
13. The inverse of the correlation is given as inv(**C**).
14. The time in seconds NOA used to construct the above design.

EXAMPLE 1, $L_4(2^3)$, m = 3 2-level factors (parameters) and n = 4 runs

A small example with m = 3 2-level factors (parameters) in n = 4 runs is used to demonstrate the output obtained with the NOA module. In the DOS directory the command C:\Gendex> NOA was used. At the prompt Choose the number of 2-level factors, the number 3 was highlighted and OK was clicked. At the prompt Choose the number of runs, the number 4 was highlighted and OK was clicked. At the prompt Choose a criterion, Booth-Cox E(s^2) was highlighted and OK was clicked. Ok was clicked on for Enter a random seed and Enter the number of tries as no random seed or number of tries was entered. The output for this design follows:

NOA 2.0: Construct orthogonal and near-orthogonal arrays with mixed levels
(C) 2001 Design Computing (URL: http://designcomputing.hypermart.net)

Note: design for m=3 and n=4.

try #	1
seed	1005078373830
# of iterations	0
ave	0*
max of s(i,j)	0
freq of max	3
D-eff	1
A-eff	1

Factor levels:

-1	1	1
1	1	-1
1	-1	1
-1	-1	-1

X'X

4	0	0
	4	0
		4

C (Correlation matrix)

1	0	0
	1	0
		1

inverse(C)

1	0	-0
	1	-0
		1

Note: NOA used 0.06 seconds.
Note: this software is licensed to AV Biometrics (Australia).

The design was obtained at the first try and requiring no iterations using the random seed 1005078373830. The matrix **X** is given under the notation Factor levels. From the **X'X** matrix, we see that all elements above the main right diagonal are zero and hence the average of their squares will be zero. An asterisk appears after the ave value 0 which indicates that this design is the best possible and has reached the lower bound. The maximum value of the s_{ij} values is 0 and this value has a frequency of $m(m - 1)/2 = 3$. The A-eff may be computed as the determinant of the correlation matrix **C** or as the determinant of the **X'X** matrix divided by the product of the main right diagonal elements. The design obtained is a saturated orthogonal main effect plan. It required only 0.06 second of computer time to create.

EXAMPLE 2, $L_5(2^8)$, m = 8 factors and n = 5 runs

Since there are more factors, m = 8, than there are number of runs, n = 5, a supersaturated fraction will be obtained. Using the steps outlined above and selecting 1123 as the random seed, the output for this design is:

NOA 2.0: Construct orthogonal and near-orthogonal arrays with mixed levels
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Note: design for m=8 and n=5.

try # 1

seed	1123
# of iterations	3
ave	3.5714*
max of s(i,j)	3
freq of max	9

Factor levels:

-1	-1	1	-1	1	1	1	-1
-1	-1	-1	1	1	-1	-1	-1
-1	1	-1	1	-1	-1	1	1
1	1	1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	1	-1	1

X'X

5	1	1	-3	-3	1	-3	1
	5	1	1	-3	-3	1	1
		5	-3	1	1	1	-3
			5	1	-3	1	1
				5	1	1	-3
					5	1	1
						5	1
							5

C (Correlation matrix)

1	0.2000	0.2000	-0.6000	-0.6000	0.2000	-0.6000	0.2000
	1	0.2000	0.2000	-0.6000	-0.6000	0.2000	0.2000
		1	-0.6000	0.2000	0.2000	0.2000	-0.6000
			1	0.2000	-0.6000	0.2000	0.2000
				1	0.2000	0.2000	-0.6000
					1	0.2000	0.2000
						1	0.2000
							1

Note: NOA used 0.16 seconds.
Note: this software is licensed to AV Biometrics (Australia).

The design was obtained at the third iteration of the first try. The design matrix **X** is given under Factor levels. The sum of squares of the s_{ij} elements above the main right diagonal of **X'X** is 100. Therefore, the average is 100/28 = 3.5714. An asterisk follows this number, indicating that the lower bound was reached and that this is the best design that can be obtained. The maximum absolute value for the s_{ij} is 3 and there are 9 such values. The D-eff and A-eff values are not listed on the output as they were for Example 1, owing to the fact that the determinants of **X'X** and of the correlation matrix are zero.

EXAMPLE 3, $L_{24}(2^{12})$, m = 12 factors and n = 24 runs

Using the above outlined steps for m = 12 factors, n = 24 runs, and a random seed of 462529680, the following design was obtained:

NOA 2.0: Construct orthogonal and near-orthogonal arrays with mixed levels

Note: design for m=12 and n=24.

try #	1
seed	462529680
# of iterations	23
ave	1.4545
max of s(i,j)	4
freq of max	6
D-eff	0.9858
A-eff	0.9714

try #	2
seed	-412023794
# of iterations	26
ave	0.9697
max of s(i,j)	4
freq of max	4
D-eff	0.9907
A-eff	0.9813

try #	24
seed	-194563892
# of iterations	22
ave	0.4848
max of s(i,j)	4
freq of max	2
D-eff	0.9953
A-eff	0.9906

Factor levels:

1	1	-1	-1	1	1	1	1	-1	-1	-1	1
1	1	1	1	1	1	1	-1	1	1	1	1
1	1	-1	-1	-1	-1	-1	1	1	1	-1	-1
-1	-1	-1	1	1	1	-1	1	1	1	1	1
1	-1	1	-1	-1	1	1	1	1	1	-1	1
-1	1	-1	-1	1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	1	1	-1	1	-1	-1	1	-1	-1
-1	1	-1	1	-1	1	-1	-1	1	1	-1	1
-1	1	1	1	-1	-1	-1	-1	-1	-1	1	1
-1	-1	1	1	-1	-1	1	1	-1	1	-1	1
1	-1	-1	1	1	-1	-1	-1	1	-1	-1	1
1	1	-1	1	-1	1	1	1	1	-1	1	-1
-1	1	1	-1	1	1	1	-1	-1	1	1	-1
-1	-1	1	-1	-1	1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1
1	-1	1	-1	-1	-1	-1	-1	1	1	1	-1
-1	1	1	1	-1	1	-1	1	-1	-1	-1	-1
1	1	-1	-1	-1	-1	1	-1	-1	1	1	1
-1	1	1	-1	1	-1	-1	1	1	-1	1	1
1	-1	-1	1	-1	1	-1	-1	-1	-1	1	-1
1	-1	1	-1	1	1	-1	-1	-1	-1	-1	1
-1	-1	-1	-1	1	1	-1	1	-1	1	1	-1

1	1	1	1	1	-1	-1	1	-1	1	-1	-1
1	-1	1	1	1	-1	1	1	1	-1	1	-1

X'X

24	0	0	0	0	0	0	0	4	0	0	0
	24	0	0	0	0	0	0	0	0	0	0
		24	0	0	0	0	0	0	0	0	0
			24	0	0	-4	0	0	0	0	0
				24	0	0	0	0	0	0	0
					24	0	0	0	0	0	0
						24	0	0	0	0	0
							24	0	0	0	0
								24	0	0	0
									24	0	0
										24	0
											24

C (Correlation matrix)

1	0	0	0	0	0	0	0	0.1667	0	0	0
	1	0	0	0	0	0	0	0	0	0	0
		1	0	0	0	0	0	0	0	0	0
			1	0	0	-0.1667	0	0	0	0	0
				1	0	0	0	0	0	0	0
					1	0	0	0	0	0	0
						1	0	0	0	0	0
							1	0	0	0	0
								1	0	0	0
									1	0	0
										1	0
											1

inverse(C)

1.0286	0	0	0	0	0	0	0	-0.1714	0	0	-0
	1	0	0	0	0	0	0	0	0	0	-0
		1	0	0	0	0	0	0	0	0	-0
			1.0286	0	0	0.1714	0	0	0	0	-0
				1	0	0	0	0	0	0	-0
					1	0	0	0	0	0	-0
						1.0286	0	0	0	0	-0
							1	0	0	0	-0
								1.0286	0	0	-0
									1	0	-0
										1	-0
											1

Note: NOA used 0.93 seconds.
Note: this software is licensed to AV Biometrics (Australia).

This design is almost an orthogonal one. The D-eff and A-eff values are very close to one. There are only two off upper diagonal elements, 4 and -4, that are non-zero. The ave value is 32/66 = 0.4848. This design required only 0.93 second of computer time and was obtained on the 22nd iteration of the 24th try.

EXAMPLE 4, $L_{20}(5 \cdot 2^8)$, m = 12 parameters n = 20 runs

Suppose that one 5-level factor is to be combined with eight 2-level factors in an orthogonal or near-orthogonal array (a nine factor main effect plan) in n = 20 runs. The first step is to create a set of 4 = 5 - 1 single degree of freedom contrasts such as Helmert, orthogonal polynomial, or some other set of contrasts. We choose the orthogonal polynomials for our example. The next step is to create an input file named NOA.TXT using NOTEPAD in DOS. The input file will have k = 4 columns and n = 20 rows. The NOA module considers columns as factors (parameters) and this will be noted in the output value for the parameter m = 4 + 8 = 12 for this example. This file is saved and then the command C:\Gendex>NOA is used to obtain the output given below. The input file is the first four columns of the design matrix given under Factor levels.

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Note: design for m=12 and n=20.

try #	1
seed	1005665348910
# of iterations	25
ave	1.4545
max of s(i,j)	4
freq of max	6
D-eff	0.9773
A-eff	0.9514

try #	2
seed	-517584022
# of iterations	16
ave	1.2121
max of s(i,j)	4
freq of max	5
D-eff	0.9842
A-eff	0.9692

try #	3
seed	-1721940523
# of iterations	19
ave	0.7273
max of s(i,j)	4
freq of max	3
D-eff	0.9894
A-eff	0.9778

try #	7
seed	1048384152
# of iterations	22
ave	0.7273
max of s(i,j)	4
freq of max	3

D-eff	0.9898
A-eff	0.9796

Factor levels:

-2	2	-1	1	1	1	1	1	-1	-1	1	1
-2	2	-1	1	1	1	-1	-1	-1	1	1	-1
-2	2	-1	1	-1	-1	-1	1	1	-1	-1	1
-2	2	-1	1	-1	-1	1	-1	1	1	-1	-1
-1	-1	2	-4	1	-1	-1	-1	-1	-1	-1	1
-1	-1	2	-4	-1	-1	1	-1	-1	1	1	-1
-1	-1	2	-4	1	1	1	1	1	-1	-1	-1
-1	-1	2	-4	-1	1	-1	1	1	1	1	1
0	-2	0	6	-1	1	1	-1	1	-1	-1	-1
0	-2	0	6	1	-1	-1	1	1	1	1	1
0	-2	0	6	-1	1	1	1	-1	1	-1	1
0	-2	0	6	1	-1	-1	-1	-1	-1	1	-1
1	-1	-2	-4	1	1	1	-1	1	1	1	1
1	-1	-2	-4	1	-1	1	1	1	-1	1	-1
1	-1	-2	-4	-1	-1	-1	1	-1	1	-1	-1
1	-1	-2	-4	-1	1	-1	-1	-1	-1	-1	1
2	2	1	1	1	-1	1	-1	1	1	-1	1
2	2	1	1	1	1	-1	1	-1	1	-1	-1
2	2	1	1	-1	-1	1	1	-1	-1	1	1
2	2	1	1	-1	1	-1	-1	1	-1	1	-1

X'X

40	0	0	0	0	0	0	0	0	0	0	0
	56	0	0	0	0	0	0	0	0	0	0
		40	0	0	0	0	0	0	0	0	0
			280	0	0	0	0	0	0	0	0
				20	0	0	0	0	0	4	0
					20	0	0	0	0	0	0
						20	0	4	0	0	0
							20	0	0	0	4
								20	0	0	0
									20	0	0
										20	0
											20

C (Correlation matrix)

1	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0
		1	0	0	0	0	0	0	0	0	0
			1	0	0	0	0	0	0	0	0
				1	0	0	0	0	0	0.2000	0
					1	0	0	0	0	0	0
						1	0	0.2000	0	0	0
							1	0	0	0	0.2000
								1	0	0	0
									1	0	0
										1	0
											1

inverse(C)

1	0	0	0	0	0	0	0	0	0	0	-0
	1	0	0	0	0	0	0	0	0	0	-0
		1	0	0	0	0	0	0	0	0	-0
			1	0	0	0	0	0	0	0	-0
				1.0417	0	0	0	0	0	-0.2083	-0
					1	0	0	0	0	0	-0
						1.0417	0	-0.2083	0	0	-0
							1.0417	0	0	0	-0.2083
								1.0417	0	0	-0
									1	0	-0
										1.0417	-0
											1.0417

Note: the first 4 columns of the design are protected columns.
Note: NOA used 0.55 seconds.
Note: this software is licensed to AV Biometrics (Australia).

The Note at the end of the output indicates that the four columns used as input are protected columns. The above plan has only three non-zero elements (4) above the main right diagonal and hence is near-orthogonal. Note that any set of contrasts may be used for the input matrix in NOA.TXT. Nguyen has shown that it is possible to obtain a diagonal matrix X'X but several tries with this version of the Gendex toolkit did not achieve this.

EXAMPLE 5, $L_{24}(2(3) \,.\, 2^8)$, m = 14 parameters and n = 24 runs

For this example, a 2-level factor by 3-level factor set of contrasts is used as the input file NOA.TXT with five single degree of freedom contrasts together with eight 2-level factors. The number of parameters m = 5 + 8 = 13. Using the command C:\Gendex> notepad NOA.TXT and answering YES to create a new file, a file named NOA.TXT with five columns and n = 24 rows is created and saved. Returning to C:\Gendex>, type NOA, answer YES to the question describing the NOA.TXT file, at the prompt, select eight 2-level factors, and n = 24 runs to obtain the following output:

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Note: design for m=13 and n=24.

try #	1
seed	1005769646440
# of iterations	38
ave	0.8205
max of s(i,j)	4
freq of max	4
D-eff	0.9912
A-eff	0.9822

try # 8

seed	-364130616
# of iterations	39
ave	0*
max of s(i,j)	0
freq of max	78
D-eff	1
A-eff	1

Factor levels:

-1	-1	1	1	-1	1	1	-1	-1	1	1	1	1
-1	-1	1	1	-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	1	1	-1	-1	-1	1	1	-1	1	1	1
-1	-1	1	1	-1	1	1	1	-1	1	-1	-1	-1
-1	0	-2	0	2	-1	-1	1	-1	-1	1	-1	-1
-1	0	-2	0	2	-1	1	-1	-1	1	-1	-1	1
-1	0	-2	0	2	1	-1	1	1	1	-1	1	-1
-1	0	-2	0	2	1	1	-1	1	-1	1	1	1
-1	1	1	-1	-1	1	-1	1	1	1	-1	-1	1
-1	1	1	-1	-1	-1	-1	1	-1	1	1	1	1
-1	1	1	-1	-1	1	1	-1	1	-1	1	-1	-1
-1	1	1	-1	-1	-1	1	-1	-1	-1	-1	1	-1
1	-1	1	-1	1	-1	1	1	1	-1	1	-1	1
1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
1	-1	1	-1	1	1	-1	-1	-1	1	1	-1	1
1	-1	1	-1	1	1	-1	1	-1	-1	-1	1	-1
1	0	-2	0	-2	1	-1	-1	1	-1	-1	1	1
1	0	-2	0	-2	1	1	1	-1	-1	1	-1	-1
1	0	-2	0	-2	-1	-1	-1	-1	1	1	1	-1
1	0	-2	0	-2	-1	1	1	1	1	-1	-1	1
1	1	1	1	1	1	1	1	-1	-1	-1	1	1
1	1	1	1	1	1	-1	-1	1	1	1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1
1	1	1	1	1	-1	1	1	1	1	1	1	-1

X'X

24	0	0	0	0	0	0	0	0	0	0	0	0
	16	0	0	0	0	0	0	0	0	0	0	0
		48	0	0	0	0	0	0	0	0	0	0
			16	0	0	0	0	0	0	0	0	0
				48	0	0	0	0	0	0	0	0
					24	0	0	0	0	0	0	0
						24	0	0	0	0	0	0
							24	0	0	0	0	0
								24	0	0	0	0
									24	0	0	0
										24	0	0
											24	0
												24

C (Correlation matrix)

1	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0	0
		1	0	0	0	0	0	0	0	0	0	0

1	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0
		1	0	0	0	0	0	0	0
			1	0	0	0	0	0	0
				1	0	0	0	0	0
					1	0	0	0	0
						1	0	0	0
							1	0	0
								1	0
									1

inverse(C)

1	0	0	0	0	0	0	0	0	0	0	0	-0
	1	0	0	0	0	0	0	0	0	0	0	-0
		1	0	0	0	0	0	0	0	0	0	-0
			1	0	0	0	0	0	0	0	0	-0
				1	0	0	0	0	0	0	0	-0
					1	0	0	0	0	0	0	-0
						1	0	0	0	0	0	-0
							1	0	0	0	0	-0
								1	0	0	0	-0
									1	0	0	-0
										1	0	-0
											1	-0
												1

Note: the first 5 columns of the design are protected columns.
Note: NOA used 0.72 seconds.
Note: this software is licensed to AV Biometrics (Australia).

The above design allows estimation of ten main effects and one 2-level by 3-level factor interaction. This design is an orthogonal one as there are only zeros in the off upper non-diagonal elements. The design required several tries before an orthogonal treatment design was obtained, illustrating the fact that it may be useful to perform several attempts for a complex design such as this one as a better one may be obtained than resulted from the first attempt. Note the asterisk beside ave indicating that this is the best design that can be obtained.

COMMENTS

When a file like NOA.TXT is created in notepad and saved in the C:\Gendex folder, a "refresh" of the files was often required in order for the last saved NOA.TXT file rather than the previous one to appear. The file can then be copied or printed. For some of the larger plans, it may be advantageous to obtain the plan sequentially by starting with a fewer number of factors and saving this plan as the input file NOA.TXT. Then the additional factors are added.

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